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Large Numbers Hypothesis and Whitrow–Randall–Sciama Relation in Brans–Dicke Theory

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It has recently been suggested in Brans–Dicke theory that the so-called Whitrow–Randall–Sciama relation plus the assumption of a constant deceleration parameter implies the large numbers hypothesis. It is shown that this claim is not always true.

There exist in nature some very large dimensionless numbers, such as the ratio of the electric to the gravitational force between a proton and an electron,

$$N_1 = \frac{e^2}{Gm_p m_e} \approx 10^{39} \tag{1}$$

or the present age of the universe expressed in units of the atomic light-crossing time,

$$N_2 = \frac{t_0}{e^2/(m_e c^3)} \approx 10^{39} \tag{2}$$

The similarity between such seemingly unrelated large numbers led Dirac (Barrow and Tipler, 1986, and references therein) to propose the "large numbers hypothesis" (LNH), which states:

> Any two of the very large dimensionless numbers occurring in nature are connected by a simple mathematical relation, in which the coefficients are of the order of magnitude unity.

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A consequence of the above is that the numbers in (1) and (2) must be equal,

$$N_1 = N_2 \tag{3}$$

Dirac then chose to satisfy (3) by the radical requirement that the gravitational "constant" is time-varying,

$$G \sim 1/t \tag{4}$$

The amount of matter in the visible universe leads to another large number,

$$N_3 = \frac{4\pi(ct)^3\varrho}{3m_p} \approx 10^{78} \tag{5}$$

where ϱ is the density of matter in the universe. By the LNH, we must have

$$\sqrt{N_3} \approx t \tag{6}$$

which leads to

$$\varrho \sim 1/t \tag{7}$$

The Whitrow–Randall–Sciama (WRS) relation, on the other hand, is given by (Berman and Som, 1990; Berman, 1992; and references therein)

$$G\varrho \sim 1/t^2 \tag{8}$$

Let q be the deceleration parameter, which is defined by

$$q = -\ddot{R}R/\dot{R}^2 \tag{9}$$

where R is the scale factor of the universe, and the dot denotes a derivative with respect to time. Now Berman (1992) has claimed that in BDT

$$\text{WRS} \wedge (q = \text{const}) \Rightarrow (4) \wedge (7) \tag{10}$$

To examine the above claim (10) in more detail, let us first consider the field equations in BDT for the Robertson–Walker metric for the zero curvature case and a perfect fluid (Weinberg, 1972),

$$\frac{\dot{R}^2}{R^2} = \frac{\varrho}{3\varphi} - \frac{\dot{\varphi}}{\varphi}\frac{\dot{R}}{R} + \frac{\omega}{6}\frac{\dot{\varphi}^2}{\varphi^2} \tag{11}$$

$$\ddot{\varphi} + 3\dot{\varphi}\frac{\dot{R}}{R} = \frac{\varrho - 3p}{3 + 2\omega} \tag{12}$$

$$\dot{\varrho} + 3\frac{\dot{R}}{R}(\varrho + p) = 0 \tag{13}$$

where φ is the BD scalar field normalized such that $8\pi G_N = 1$ (G_N is the present value of G), ω is the BD coupling parameter, and p is the cosmic pressure. In BDT, G varies as the inverse of φ. Assuming the usual equation of state of the form

$$p = (\gamma - 1)\varrho \tag{14}$$

we have that equations (12) and (13) become

$$\ddot{\varphi} + 3\dot{\varphi}\frac{\dot{R}}{R} = \frac{4 - 3\gamma}{3 + 2\omega}\varrho \tag{15}$$

$$\dot{\varrho} + 3\gamma\varrho\frac{\dot{R}}{R} = 0 \tag{16}$$

It immediately follows from equation (16) that $\varrho = M/R^{3\gamma}$, $M = \text{const}$.

Lorenz–Petzold (1984) has given a set of power-law solutions to equations (11), (15), and (16) due to Nariai (1968):

$$R = at^s \tag{17}$$

$$\varphi = bt^v \tag{18}$$

$$s = \frac{2 + 2\omega(2 - \gamma)}{4 + 3\omega\gamma(2 - \gamma)} \tag{19}$$

$$v = \frac{2(4 - 3\gamma)}{4 + 3\omega\gamma(2 - \gamma)} \tag{20}$$

$$\frac{M}{ba^{3\gamma}} = \frac{3 + 2\omega}{4 - 3\gamma}[v(v - 1) + 3sv] \tag{21}$$

We notice that all these solutions have constant deceleration parameter.

To show that Berman's claim is false, consider the following particular solutions from equations (17)–(21) obtained by putting $\omega = 9$ [motivation for this choice of ω is given by Liddle *et al.* (1992)] and $\gamma = 1$:

$$s = 20/31, \qquad v = 2/31$$

It is straightforward to verify that the WRS relation is also satisfied,

$$G\varrho \sim \frac{1}{\varphi}\frac{1}{R^{3\gamma}} \sim \frac{1}{t^2}$$

but

$$\varrho \sim \frac{1}{t^{60/31}}$$

$$G \sim \frac{1}{t^{2/31}}$$

and thus Berman's claim is false.

In fact, we may conjecture that, within the space of all solutions to equations (11), (15), and (16) for which $q = \text{const}$ and for which the WRS relation is true, the set for which claim (10) holds is one of zero measure.

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